ME 141
Engineering Mechanics

Portion 8
Introduction to Dynamics: Kinematics

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Dynamics

Kinematics:
Deals with the motion (position, velocity, acceleration, time) of a body without considering the cause of motion (Force)

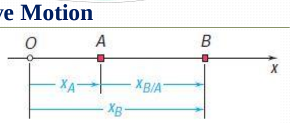
- Kinematics of a point (The body is treated as a point whose rotation about its own center can be neglected)
- Kinematics of a rigid body (The body is treated as a point whose rotation about its own center can not be neglected)

Kinetics:
Deals with the motion (position, velocity, acceleration, time) of a body without considering the cause of motion (Force)

- Kinetics of a point (The body is treated as a point whose rotation about its own center can be neglected)
- Kinetics of a rigid body (The body is treated as a point whose rotation about its own center can not be neglected)

Kinematics of a Point:
Relative Motion

Suppose, **O** is any reference point.
The relative position of **B** with respect to **A** is given by,



$$x_{B/A} = x_B - x_A$$

If it is divided by t, then similar formula can found for velocity.

$$v_{B/A} = v_B - v_A$$

Similarly, for acceleration,

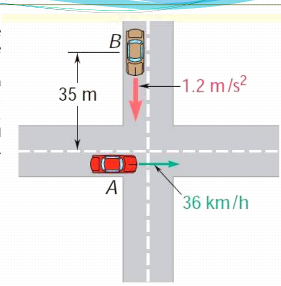
$$a_{B/A} = a_B - a_A$$

In vector form,

$$\mathbf{X}_{B/A} = \mathbf{X}_B - \mathbf{X}_A$$
$$\mathbf{V}_{B/A} = \mathbf{V}_B - \mathbf{V}_A$$
$$\mathbf{a}_{B/A} = \mathbf{a}_B - \mathbf{a}_A$$

Problem 8.1 (Beer Johnston_10th edition_Ex11.9)

Automobile A is traveling east at the constant speed of 36 km/h. As automobile A crosses the intersection shown, automobile B starts from rest 35 m north of the intersection and moves south with a constant acceleration of 1.2 m/s². Determine the position, velocity, and acceleration of B relative to A, 5 s after A crosses the intersection.



Kinematics of a Point:
Dependent Motion

Total Length of the Rope is constant during the motion of any loads or pulleys.

Neglecting the distance from the support and load to the pulleys center, we can write,

$$x_A + 2x_B = \text{constant} \text{ ----(1)}$$

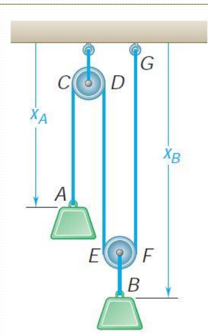
So, any small disturbance Δx_A will cause the disturbance Δx_B according to the formula.
So, $\Delta x_B = -0.5\,\Delta x_A$

Differentiating equation (1) wrt t,

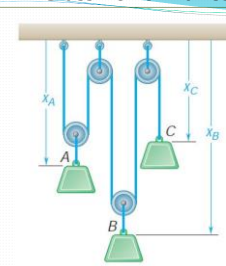
$$v_A + 2v_B = 0 \text{ -----(2)}$$

Similarly,

$$a_A + 2a_B = 0 \text{ -----(3)}$$



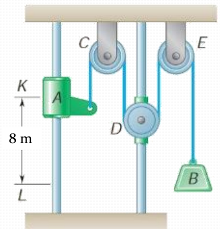
Class Performance



Position Equation: $2x_A + 2x_B + x_C = \text{constant}$
Velocity Equation: $2v_A + 2v_B + v_C = 0$
Acceleration Equation: $2a_A + 2a_B + a_C = 0$

Problem 8.2 (Beer Johnston_10th edition_Ex11.5)

Collar *A* and block *B* are connected by a cable passing over three pulleys *C*, *D*, and *E* as shown. Pulleys *C* and *E* are fixed, while *D* is attached to a collar which is pulled downward with a constant velocity of 3 m/s. At t = 0, collar *A* starts moving downward from position *K* with a constant acceleration and no initial velocity. Knowing that the velocity of collar *A* is 12 m/s as it passes through point *L*, determine the change in elevation, the velocity, and the acceleration of block *B* when collar *A* passes through *L*.



Kinematics of a Point:

Curvilinear Motion

Tangential and Normal Components

Velocity:

Tangential Component of Velocity, $v_t = v$

There is no normal velocity component in curved motion.

Hence, Normal Component of Velocity, $v_n = 0$

Velocity, $\mathbf{v} = v_t\, \boldsymbol{e_t} + v_n\, \boldsymbol{e_n} = v\, \boldsymbol{e_t}$

Here, $\boldsymbol{e_t}$ is the unit vector in tangential direction and

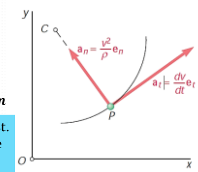
$\boldsymbol{e_n}$ is the unit vector in normal direction of the curved motion.

Acceleration:

Tangential Component of Acceleration, $a_t = \frac{dv}{dt}$

Normal Component of Acceleration, $a_n = \frac{v^2}{\rho}$

Acceleration, $\mathbf{a} = a_t\, \boldsymbol{e_t} + a_n\, \boldsymbol{e_n} = \frac{dv}{dt}\boldsymbol{e_t} + \frac{v^2}{\rho}\boldsymbol{e_n}$



ρ is the radius of curvature. For circular motion, ρ = const. Normal Component of acceleration describes the change in direction of motion while tangential component indicates the change in the speed.

Kinematics of a Point:

Curvilinear Motion

Radial and Transverse Components

Velocity:

Radial Component of Velocity, $v_r = \dot{r}$

Hence, Normal Component of Velocity, $v_\theta = r\,\dot{\theta}$

Velocity, $\mathbf{v} = v_r\, \boldsymbol{e_r} + v_\theta\, \boldsymbol{e_\theta} = \dot{r}\, \boldsymbol{e_r} + r\,\dot{\theta}\, \boldsymbol{e_\theta}$

Here, $\boldsymbol{e_r}$ is the unit vector in Radial direction and

$\boldsymbol{e_\theta}$ is the unit vector in transverse direction of the curve motion.

Acceleration:

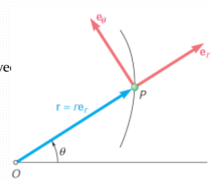
Radial Component of Acceleration, $a_r = \ddot{r} - r\,\dot{\theta}^2$

Transverse Component of Acceleration, $a_\theta = r\ddot{\theta} + 2\,\dot{r}\dot{\theta}$

Acceleration, $\mathbf{a} = a_r\, \boldsymbol{e_r} + a_\theta\, \boldsymbol{e_\theta}$

$= (\ddot{r} - r\,\dot{\theta}^2)\, \boldsymbol{e_r} + (r\ddot{\theta} + 2\,\dot{r}\dot{\theta})\, \boldsymbol{e_\theta}$

r is the radius of curvature.



What will be the components in case of circular motion???

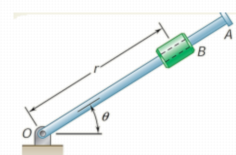
Problem 8.3 (Beer Johnston_10th edition Ex 11.12)

The rotation of the 0.9 m arm *OA* about *O* is defined by the relation $\theta = 0.15\,t^2$, where θ is expressed in radians and t in seconds. Collar *B* slides along the arm in such a way that its distance from O is $r = 0.9 - 0.12\ t^2$, where r is expressed in meters and t in seconds. After the arm *OA* has rotated through 30°, determine

(a) the total velocity of the collar,

(b) the total acceleration of the collar,

(c) the relative acceleration of the collar with respect to the arm.



References

➢**Vector Mechanics for Engineers: Statics and Dynamics**
Ferdinand Beer, Jr., E. Russell Johnston, David Mazurek, Phillip Cornwell.